Mathematica 11.3 Integration Test Results

Test results for the 111 problems in "1.2.2.5 P(x) (a+b x^2+c x^4)^p.m"

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x}{1+x^2+x^4}\,dx$$

Optimal (type 3, 92 leaves, 15 steps):

$$-\frac{d\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{d\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{e\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{4}d\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}d\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 98 leaves):

$$\frac{1}{6}\,i\left(\sqrt{6-6\,i\sqrt{3}}\;d\,\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]-\sqrt{6+6\,i\sqrt{3}}\;d\,\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]+2\,i\sqrt{3}\;e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]\right)$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2}{1+x^2+x^4}\,dx$$

Optimal (type 3, 104 leaves, 14 steps):

$$-\frac{(d+f)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(d+f)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{e\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{4}(d-f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}(d-f)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 121 leaves):

$$\frac{\left(2\,i\,d+\left(-i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{\sqrt{6+6\,i\sqrt{3}}}+\frac{\left(-2\,i\,d+\left(i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{\sqrt{6-6\,i\sqrt{3}}}-\frac{e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]}{\sqrt{3}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2+g\,x^3}{1+x^2+x^4}\,dx$$

Optimal (type 3, 127 leaves, 15 steps):

$$-\frac{(d+f)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(d+f)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(2e-g)\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{4}(d-f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}(d-f)\,\text{Log}\left[1+x+x^2\right]+\frac{1}{4}g\,\text{Log}\left[1+x^2+x^4\right]$$

Result (type 3, 150 leaves):

$$\frac{1}{8\sqrt{3}}\left(2\sqrt{2-2i\sqrt{3}}\left(2i\,d+\left(-i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]+2\left(\sqrt{2+2i\sqrt{3}}\left(-2i\,d+\left(i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]+(-4e+2g)\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]+\sqrt{3}\,g\,\text{Log}\left[1+x^2+x^4\right]\right)\right)$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2+g\,x^3+h\,x^4}{1+x^2+x^4}\,dx$$

Optimal (type 3, 136 leaves, 17 steps):

$$h\,x-\frac{(d+f-2h)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(d+f-2h)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}}+\frac{(2e-g)\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{2\sqrt{3}}-\frac{1}{4}(d-f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{4}(d-f)\,\text{Log}\left[1+x+x^2\right]+\frac{1}{4}g\,\text{Log}\left[1+x^2+x^4\right]$$

Result (type 3, 165 leaves):

$$\frac{1}{24}\left(24\,h\,x+4\left(\left(3i+\sqrt{3}\right)d+\left(-3i+\sqrt{3}\right)f-2\sqrt{3}\,h\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]+4\left(\left(-3i+\sqrt{3}\right)d+\left(3i+\sqrt{3}\right)f-2\sqrt{3}\,h\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]-8\sqrt{3}\,e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]+4\sqrt{3}\,g\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]+6\,g\,\text{Log}\left[1+x^2+x^4\right]\right)$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2+g\,x^3+h\,x^4+i\,x^5}{1+x^2+x^4}\,dx$$

Optimal (type 3, 151 leaves, 19 steps):

$$h x+\frac{i x^2}{2}-\frac{(d+f-2 h) \text{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{2 \sqrt{3}}+\frac{(d+f-2 h) \text{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{2 \sqrt{3}}+\frac{(2 e-g-i) \text{ArcTan}\left[\frac{1+2 x^2}{\sqrt{3}}\right]}{2 \sqrt{3}}-\frac{1}{4} (d-f) \text{Log}\left[1-x+x^2\right]+\frac{1}{4} (d-f) \text{Log}\left[1+x+x^2\right]+\frac{1}{4} (g-i) \text{Log}\left[1+x^2+x^4\right]$$

Result (type 3, 187 leaves):

$$\frac{1}{12} \left(6 x (2 h+i x)+\left(1+\mathrm{i} \sqrt{3}\right) \left(2 \sqrt{3} d-\left(3 \mathrm{i}+\sqrt{3}\right) f-\left(-3 \mathrm{i}+\sqrt{3}\right) h\right) \text{ArcTan}\left[\frac{1}{2} \left(-\mathrm{i}+\sqrt{3}\right) x\right]+\left(\mathrm{i}+\sqrt{3}\right) \left(-2 \mathrm{i} \sqrt{3} d+\left(3+\mathrm{i} \sqrt{3}\right) f+\mathrm{i} \left(3 \mathrm{i}+\sqrt{3}\right) h\right) \text{ArcTan}\left[\frac{1}{2} \left(\mathrm{i}+\sqrt{3}\right) x\right]-2 \sqrt{3} (2 e-g-i) \text{ArcTan}\left[\frac{\sqrt{3}}{1+2 x^2}\right]+3 (g-i) \text{Log}\left[1+x^2+x^4\right]\right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x}{\left(1+x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 140 leaves, 17 steps):

$$\frac{d x \left(1-x^2\right)}{6 \left(1+x^2+x^4\right)}+\frac{e \left(1+2 x^2\right)}{6 \left(1+x^2+x^4\right)}-\frac{d \text{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{3 \sqrt{3}}+\frac{d \text{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{3 \sqrt{3}}+\frac{2 e \text{ArcTan}\left[\frac{1+2 x^2}{\sqrt{3}}\right]}{3 \sqrt{3}}-\frac{1}{4} d \text{Log}\left[1-x+x^2\right]+\frac{1}{4} d \text{Log}\left[1+x+x^2\right]$$

Result (type 3, 146 leaves):

$$\frac{e+2 e x^2+d \left(x-x^3\right)}{6 \left(1+x^2+x^4\right)}-\frac{\left(-11 \mathrm{i}+\sqrt{3}\right) d \text{ArcTan}\left[\frac{1}{2} \left(-\mathrm{i}+\sqrt{3}\right) x\right]}{6 \sqrt{6+6 \mathrm{i} \sqrt{3}}}-\frac{\left(11 \mathrm{i}+\sqrt{3}\right) d \text{ArcTan}\left[\frac{1}{2} \left(\mathrm{i}+\sqrt{3}\right) x\right]}{6 \sqrt{6-6 \mathrm{i} \sqrt{3}}}-\frac{2 e \text{ArcTan}\left[\frac{\sqrt{3}}{1+2 x^2}\right]}{3 \sqrt{3}}$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x+f x^2}{\left(1+x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 165 leaves, 16 steps):

$$\frac{e\,(1+2x^2)}{6\,(1+x^2+x^4)}+\frac{x\,(d+f-(d-2f)\,x^2)}{6\,(1+x^2+x^4)}-\frac{(4d+f)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{(4d+f)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{12\sqrt{3}}+$$
$$\frac{2\,e\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{8}\,(2d-f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{8}\,(2d-f)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 186 leaves) :

$$\frac{1}{36}\left(\frac{6\,(e+2\,e\,x^2+x\,(d+f-d\,x^2+2\,f\,x^2))}{1+x^2+x^4}-\right.$$
$$\frac{\left(\left(-11\,i+\sqrt{3}\right)d-2\left(-2\,i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-$$
$$\left.\frac{\left(\left(11\,i+\sqrt{3}\right)d-2\left(2\,i+\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-8\sqrt{3}\,e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]\right)$$

Problem 33: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3}{(1+x^2+x^4)^2}\,dx$$

Optimal (type 3, 179 leaves, 15 steps) :

$$\frac{x\,(d+f-(d-2f)\,x^2)}{6\,(1+x^2+x^4)}+\frac{e-2g+(2e-g)\,x^2}{6\,(1+x^2+x^4)}-\frac{(4d+f)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{(4d+f)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{12\sqrt{3}}+$$
$$\frac{(2e-g)\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{8}\,(2d-f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{8}\,(2d-f)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 200 leaves) :

$$\frac{1}{36}\left(\frac{6\left(e+2\,e\,x^2-g\,\left(2+x^2\right)+x\,\left(d+f-d\,x^2+2\,f\,x^2\right)\right)}{1+x^2+x^4}-\frac{\left(\left(-11\,\mathrm{i}+\sqrt{3}\right)\,d-2\,\left(-2\,\mathrm{i}+\sqrt{3}\right)\,f\right)\,\text{ArcTan}\left[\frac{1}{2}\,\left(-\mathrm{i}+\sqrt{3}\right)\,x\right]}{\sqrt{\frac{1}{6}\,\left(1+\mathrm{i}\,\sqrt{3}\right)}}-\frac{\left(\left(11\,\mathrm{i}+\sqrt{3}\right)\,d-2\,\left(2\,\mathrm{i}+\sqrt{3}\right)\,f\right)\,\text{ArcTan}\left[\frac{1}{2}\,\left(\mathrm{i}+\sqrt{3}\right)\,x\right]}{\sqrt{\frac{1}{6}\,\left(1-\mathrm{i}\,\sqrt{3}\right)}}-4\,\sqrt{3}\,\left(2\,e-g\right)\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2\,x^2}\right]\right)$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3+h\,x^4}{\left(1+x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 187 leaves, 15 steps):

$$\frac{e-2\,g+\left(2\,e-g\right)\,x^2}{6\,\left(1+x^2+x^4\right)}+\frac{x\,\left(d+f-2\,h-\left(d-2\,f+h\right)\,x^2\right)}{6\,\left(1+x^2+x^4\right)}-\frac{\left(4\,d+f+h\right)\,\text{ArcTan}\left[\frac{1-2\,x}{\sqrt{3}}\right]}{12\,\sqrt{3}}+\frac{\left(4\,d+f+h\right)\,\text{ArcTan}\left[\frac{1+2\,x}{\sqrt{3}}\right]}{12\,\sqrt{3}}+\frac{\left(2\,e-g\right)\,\text{ArcTan}\left[\frac{1+2\,x^2}{\sqrt{3}}\right]}{3\,\sqrt{3}}-\frac{1}{8}\,\left(2\,d-f+h\right)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{8}\,\left(2\,d-f+h\right)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 234 leaves):

$$\frac{1}{36}\left(-\frac{1}{1+x^2+x^4}6\left(g\left(2+x^2\right)-e\left(1+2x^2\right)+x\left(d\left(-1+x^2\right)+h\left(2+x^2\right)-f\left(1+2x^2\right)\right)\right)-\right.$$

$$\frac{1}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\left(\left(-11i+\sqrt{3}\right)d-2\left(-2i+\sqrt{3}\right)f+\left(-5i+\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]-$$

$$\frac{1}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}\left(\left(11i+\sqrt{3}\right)d-2\left(2i+\sqrt{3}\right)f+\left(5i+\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]-$$

$$\left.4\sqrt{3}\left(2e-g\right)\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]\right)$$

Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e x+f x^2+g x^3+h x^4+i x^5}{\left(1+x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 194 leaves, 16 steps):

$$\frac{x\left(d+f-2h-\left(d-2f+h\right)x^2\right)}{6\left(1+x^2+x^4\right)}+\frac{e-2g+i+\left(2e-g-i\right)x^2}{6\left(1+x^2+x^4\right)}-$$

$$\frac{\left(4d+f+h\right)\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{\left(4d+f+h\right)\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{12\sqrt{3}}+\frac{\left(2e-g+2i\right)\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-$$

$$\frac{1}{8}\left(2d-f+h\right)\text{Log}\left[1-x+x^2\right]+\frac{1}{8}\left(2d-f+h\right)\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 243 leaves):

$$\frac{1}{36}\left(\frac{1}{1+x^2+x^4}6\left(e+i+d x+f x-2 h x+2 e x^2-i x^2-d x^3+2 f x^3-h x^3-g\left(2+x^2\right)\right)-\right.$$

$$\frac{1}{\sqrt{\frac{1}{6}\left(1+\mathrm{i}\sqrt{3}\right)}}\left(\left(-11\,\mathrm{i}+\sqrt{3}\right)d-2\left(-2\,\mathrm{i}+\sqrt{3}\right)f+\left(-5\,\mathrm{i}+\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(-\mathrm{i}+\sqrt{3}\right)x\right]-$$

$$\frac{1}{\sqrt{\frac{1}{6}\left(1-\mathrm{i}\sqrt{3}\right)}}\left(\left(11\,\mathrm{i}+\sqrt{3}\right)d-2\left(2\,\mathrm{i}+\sqrt{3}\right)f+\left(5\,\mathrm{i}+\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(\mathrm{i}+\sqrt{3}\right)x\right]-$$

$$\left.4\sqrt{3}\left(2 e-g+2 i\right)\text{ArcTan}\left[\frac{\sqrt{3}}{1+2 x^2}\right]\right)$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e x}{\left(1+x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 185 leaves, 19 steps):

$$\frac{d x\left(1-x^2\right)}{12\left(1+x^2+x^4\right)^2}+\frac{e\left(1+2 x^2\right)}{12\left(1+x^2+x^4\right)^2}+\frac{d x\left(2-7 x^2\right)}{24\left(1+x^2+x^4\right)}+\frac{e\left(1+2 x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{13 d\,\text{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{48\sqrt{3}}+$$

$$\frac{13 d\,\text{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{48\sqrt{3}}+\frac{2 e\,\text{ArcTan}\left[\frac{1+2 x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{9}{32}d\,\text{Log}\left[1-x+x^2\right]+\frac{9}{32}d\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 186 leaves):

$$\frac{1}{144}\left(\frac{6\left(d\,x\,\left(2-7\,x^2\right)+e\,\left(4+8\,x^2\right)\right)}{1+x^2+x^4}+\frac{12\,\left(e+2\,e\,x^2+d\,\left(x-x^3\right)\right)}{\left(1+x^2+x^4\right)^2}-\frac{\left(-47\,i+7\,\sqrt{3}\right)\,d\,\text{ArcTan}\left[\frac{1}{2}\,\left(-i+\sqrt{3}\right)\,x\right]}{\sqrt{\frac{1}{6}\,\left(1+i\,\sqrt{3}\right)}}-\frac{\left(47\,i+7\,\sqrt{3}\right)\,d\,\text{ArcTan}\left[\frac{1}{2}\,\left(i+\sqrt{3}\right)\,x\right]}{\sqrt{\frac{1}{6}\,\left(1-i\,\sqrt{3}\right)}}-32\,\sqrt{3}\,e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2\,x^2}\right]\right)$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2}{\left(1+x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 223 leaves, 18 steps):

$$\frac{e\,\left(1+2\,x^2\right)}{12\,\left(1+x^2+x^4\right)^2}+\frac{x\,\left(d+f-\left(d-2\,f\right)\,x^2\right)}{12\,\left(1+x^2+x^4\right)^2}+\frac{e\,\left(1+2\,x^2\right)}{6\,\left(1+x^2+x^4\right)}+\frac{x\,\left(2\,d+3\,f-7\,\left(d-f\right)\,x^2\right)}{24\,\left(1+x^2+x^4\right)}-\frac{\left(13\,d+2\,f\right)\,\text{ArcTan}\left[\frac{1-2\,x}{\sqrt{3}}\right]}{48\,\sqrt{3}}+\frac{\left(13\,d+2\,f\right)\,\text{ArcTan}\left[\frac{1+2\,x}{\sqrt{3}}\right]}{48\,\sqrt{3}}+\frac{2\,e\,\text{ArcTan}\left[\frac{1+2\,x^2}{\sqrt{3}}\right]}{3\,\sqrt{3}}-\frac{1}{32}\,\left(9\,d-4\,f\right)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{32}\,\left(9\,d-4\,f\right)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 235 leaves):

$$\frac{1}{144}\left(\frac{6\,\left(2\,d\,x+3\,f\,x-7\,d\,x^3+7\,f\,x^3+e\,\left(4+8\,x^2\right)\right)}{1+x^2+x^4}+\frac{12\,\left(e+2\,e\,x^2+x\,\left(d+f-d\,x^2+2\,f\,x^2\right)\right)}{\left(1+x^2+x^4\right)^2}-\frac{\left(\left(-47\,i+7\,\sqrt{3}\right)\,d+\left(17\,i-7\,\sqrt{3}\right)\,f\right)\,\text{ArcTan}\left[\frac{1}{2}\,\left(-i+\sqrt{3}\right)\,x\right]}{\sqrt{\frac{1}{6}\,\left(1+i\,\sqrt{3}\right)}}-\frac{\left(\left(47\,i+7\,\sqrt{3}\right)\,d-\left(17\,i+7\,\sqrt{3}\right)\,f\right)\,\text{ArcTan}\left[\frac{1}{2}\,\left(i+\sqrt{3}\right)\,x\right]}{\sqrt{\frac{1}{6}\,\left(1-i\,\sqrt{3}\right)}}-32\,\sqrt{3}\,e\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2\,x^2}\right]\right)$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2+g\,x^3}{\left(1+x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 243 leaves, 17 steps):

$$\frac{x\left(d+f-(d-2f)\,x^2\right)}{12\left(1+x^2+x^4\right)^2}+\frac{e-2g+(2e-g)\,x^2}{12\left(1+x^2+x^4\right)^2}+\frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)}+\frac{x\left(2d+3f-7(d-f)\,x^2\right)}{24\left(1+x^2+x^4\right)}-\frac{(13d+2f)\,\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{48\sqrt{3}}+\frac{(13d+2f)\,\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{48\sqrt{3}}+\frac{(2e-g)\,\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{32}(9d-4f)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{32}(9d-4f)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 259 leaves):

$$\frac{1}{144}\left(\frac{6\left(2dx+3fx-7dx^3+7fx^3-2g\left(1+2x^2\right)+e\left(4+8x^2\right)\right)}{1+x^2+x^4}+\frac{12\left(e+2ex^2-g\left(2+x^2\right)+x\left(d+f-dx^2+2fx^2\right)\right)}{\left(1+x^2+x^4\right)^2}-\frac{\left(\left(-47i+7\sqrt{3}\right)d+\left(17i-7\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(47i+7\sqrt{3}\right)d-\left(17i+7\sqrt{3}\right)f\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-16\sqrt{3}\,(2e-g)\,\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]\right)$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\,x+f\,x^2+g\,x^3+h\,x^4}{\left(1+x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 263 leaves, 17 steps):

$$\frac{e-2g+(2e-g)x^2}{12(1+x^2+x^4)^2}+\frac{x(d+f-2h-(d-2f+h)x^2)}{12(1+x^2+x^4)^2}+\frac{(2e-g)(1+2x^2)}{12(1+x^2+x^4)}+\frac{x(2d+3f-h-(7d-7f+4h)x^2)}{24(1+x^2+x^4)}-\frac{(13d+2f+h)\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{48\sqrt{3}}+\frac{(13d+2f+h)\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{48\sqrt{3}}+\frac{(2e-g)\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{32}(9d-4f+3h)\text{Log}\left[1-x+x^2\right]+\frac{1}{32}(9d-4f+3h)\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 303 leaves):

$$\frac{1}{144}\left(-\frac{1}{1+x^2+x^4}6\left(-4e(1+2x^2)+g(2+4x^2)+x(-2d-3f+h+7dx^2-7fx^2+4hx^2)\right)+\frac{12(e+2ex^2-g(2+x^2)+x(d+f-dx^2+2fx^2-h(2+x^2)))}{(1+x^2+x^4)^2}-\frac{1}{\sqrt{\frac{1}{6}(1+i\sqrt{3})}}\left(\left(-47i+7\sqrt{3}\right)d+\left(17i-7\sqrt{3}\right)f+2\left(-7i+2\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]-\frac{1}{\sqrt{\frac{1}{6}(1-i\sqrt{3})}}\left(\left(47i+7\sqrt{3}\right)d-\left(17i+7\sqrt{3}\right)f+2\left(7i+2\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]-16\sqrt{3}(2e-g)\text{ArcTan}\left[\frac{\sqrt{3}}{1+2x^2}\right]\right)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx$$

Optimal (type 3, 269 leaves, 18 steps):

$$\frac{x(d+f-2h-(d-2f+h)x^2)}{12(1+x^2+x^4)^2}+\frac{e-2g+i+(2e-g-i)x^2}{12(1+x^2+x^4)^2}+\frac{(2e-g+i)(1+2x^2)}{12(1+x^2+x^4)}+\frac{x(2d+3f-h-(7d-7f+4h)x^2)}{24(1+x^2+x^4)}-\frac{(13d+2f+h)\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{48\sqrt{3}}+\frac{(13d+2f+h)\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{48\sqrt{3}}+\frac{(2e-g+i)\text{ArcTan}\left[\frac{1+2x^2}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{32}(9d-4f+3h)\text{Log}\left[1-x+x^2\right]+\frac{1}{32}(9d-4f+3h)\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 325 leaves):

$$\frac{1}{144}\left(\frac{1}{\left(1+x^2+x^4\right)^2}12\left(e+i+d\,x+f\,x-2\,h\,x+2\,e\,x^2-i\,x^2-d\,x^3+2\,f\,x^3-h\,x^3-g\left(2+x^2\right)\right)+\frac{1}{1+x^2+x^4}\right.$$
$$6\left(2\,i+2\,d\,x+3\,f\,x-h\,x+4\,i\,x^2-7\,d\,x^3+7\,f\,x^3-4\,h\,x^3-2\,g\left(1+2\,x^2\right)+e\left(4+8\,x^2\right)\right)-$$
$$\frac{1}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}$$
$$\left(\left(-47\,i+7\sqrt{3}\right)d+\left(17\,i-7\sqrt{3}\right)f+2\left(-7\,i+2\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right)x\right]-$$
$$\frac{1}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}\left(\left(47\,i+7\sqrt{3}\right)d-\left(17\,i+7\sqrt{3}\right)f+2\left(7\,i+2\sqrt{3}\right)h\right)\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right)x\right]-$$
$$\left.16\sqrt{3}\left(2\,e-g+i\right)\text{ArcTan}\left[\frac{\sqrt{3}}{1+2\,x^2}\right]\right)$$

Problem 103: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\left(d+e\,x+f\,x^2+g\,x^3\right)\left(a+b\,x^2+c\,x^4\right)^{3/2}dx$$

Optimal (type 4, 717 leaves, 12 steps):

$$-\left(\left(\left(18 b^3 c d-144 a b c^2 d-8 b^4 f+57 a b^2 c f-84 a^2 c^2 f\right) x \sqrt{a+b x^2+c x^4}\right) \Big/ \left(315 c^{5/2}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\right)\right)-\frac{3\left(b^2-4 a c\right)\left(2 c e-b g\right)\left(b+2 c x^2\right) \sqrt{a+b x^2+c x^4}}{256 c^3}+\frac{1}{315 c^2}$$

$$x\left(9 b^2 c d+90 a c^2 d-4 b^3 f+9 a b c f+3 c\left(9 b c d-4 b^2 f+14 a c f\right) x^2\right) \sqrt{a+b x^2+c x^4}+$$

$$\frac{\left(2 c e-b g\right)\left(b+2 c x^2\right)\left(a+b x^2+c x^4\right)^{3/2}}{32 c^2}+\frac{x\left(3\left(3 c d+b f\right)+7 c f x^2\right)\left(a+b x^2+c x^4\right)^{3/2}}{63 c}+$$

$$\frac{g\left(a+b x^2+c x^4\right)^{5/2}}{10 c}+\frac{3\left(b^2-4 a c\right)^2\left(2 c e-b g\right) \text{ArcTanh}\left[\frac{b+2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2+c x^4}}\right]}{512 c^{7/2}}+$$

$$\left(a^{1/4}\left(18 b^3 c d-144 a b c^2 d-8 b^4 f+57 a b^2 c f-84 a^2 c^2 f\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(315 c^{11/4} \sqrt{a+b x^2+c x^4}\right)-$$

$$\left(a^{1/4}\left(18 b^3 c d-144 a b c^2 d-8 b^4 f+57 a b^2 c f-84 a^2 c^2 f+\right.\right.$$

$$\left.\sqrt{a} \sqrt{c}\left(9 b^2 c d-180 a c^2 d-4 b^3 f+24 a b c f\right)\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(630 c^{11/4} \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 2588 leaves):

$$\frac{1}{161280\, c^{7/2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}}$$

$$\left(-2 \sqrt{c} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\left(a+b x^2+c x^4\right)\left(-945 b^4 g+2 b^3 c\left(945 e+x\left(512 f+315 g x\right)\right)-\right.\right.$$

$$12 b^2 c\left(-525 a g+c x\left(192 d+105 e x+64 f x^2+42 g x^3\right)\right)-$$

$$8 b c^2\left(3 a\left(525 e+256 f x+147 g x^2\right)+2 c x^3\left(1152 d+945 e x+800 f x^2+693 g x^3\right)\right)-$$

$$16 c^2\left(504 a^2 g+2 c^2 x^5\left(360 d+7 x\left(45 e+40 f x+36 g x^2\right)\right)+\right.$$

$$\left.\left.a c x\left(2160 d+7 x\left(225 e+16 x\left(11 f+9 g x\right)\right)\right)\right)\right)+$$

$$2304\, i \sqrt{2}\, b^3 c^{3/2}\left(b-\sqrt{b^2-4 a c}\right) d \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$18432\,i\,\sqrt{2}\,a\,b\,c^{5/2}\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$7296\,i\,\sqrt{2}\,a\,b^2\,c^{3/2}\left(b-\sqrt{b^2-4\,a\,c}\right)f\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$1024\,i\,\sqrt{2}\,b^4\,\sqrt{c}\,\left(-b+\sqrt{b^2-4\,a\,c}\right)f\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$10752\,i\,\sqrt{2}\,a^2\,c^{5/2}\left(-b+\sqrt{b^2-4\,a\,c}\right)f\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)+$$

$$2304\, i \sqrt{2}\, a\, b^2 c^{5/2} d \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] -$$

$$46080\, i \sqrt{2}\, a^2 c^{7/2} d \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] -$$

$$1024\, i \sqrt{2}\, a\, b^3 c^{3/2} f \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] +$$

$$6144\, i \sqrt{2}\, a^2 b\, c^{5/2} f \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}$$

$$\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] +$$

$$1890\, b^4 c \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\, e \sqrt{a+bx^2+cx^4}\, \text{Log}\left[b+2cx^2+2\sqrt{c}\sqrt{a+bx^2+cx^4}\right] -$$

$$15120\, a\, b^2 c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\, e \sqrt{a+bx^2+cx^4}\, \text{Log}\left[b+2cx^2+2\sqrt{c}\sqrt{a+bx^2+cx^4}\right] +$$

$$30240\, a^2 c^3 \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\, e \sqrt{a+bx^2+cx^4}\, \text{Log}\left[b+2cx^2+2\sqrt{c}\sqrt{a+bx^2+cx^4}\right] -$$

$$945\, b^5 \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\, g \sqrt{a+bx^2+cx^4}\, \text{Log}\left[b+2cx^2+2\sqrt{c}\sqrt{a+bx^2+cx^4}\right] +$$

$$7560\, a\, b^3 c \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\, g \sqrt{a+bx^2+cx^4}\, \text{Log}\left[b+2cx^2+2\sqrt{c}\sqrt{a+bx^2+cx^4}\right] -$$

$$\left.15120\, a^2 b\, c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\, g \sqrt{a+bx^2+cx^4}\, \text{Log}\left[b+2cx^2+2\sqrt{c}\sqrt{a+bx^2+cx^4}\right]\right)$$

Problem 104: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \left(d + e x + f x^2 + g x^3\right) \sqrt{a + b x^2 + c x^4}\, dx$$

Optimal (type 4, 505 leaves, 10 steps):

$$\frac{\left(5 b c d - 2 b^2 f + 6 a c f\right) x \sqrt{a + b x^2 + c x^4}}{15 c^{3/2} \left(\sqrt{a} + \sqrt{c}\, x^2\right)} + \frac{\left(2 c e - b g\right) \left(b + 2 c x^2\right) \sqrt{a + b x^2 + c x^4}}{16 c^2} + \frac{x \left(5 c d + b f + 3 c f x^2\right) \sqrt{a + b x^2 + c x^4}}{15 c} + \frac{g \left(a + b x^2 + c x^4\right)^{3/2}}{6 c} - \frac{\left(b^2 - 4 a c\right) \left(2 c e - b g\right) \text{ArcTanh}\left[\frac{b+2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2+c x^4}}\right]}{32 c^{5/2}} - \left(a^{1/4} \left(5 b c d - 2 b^2 f + 6 a c f\right) \left(\sqrt{a} + \sqrt{c}\, x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c}\, x^2\right)^2}}\; \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(15 c^{7/4} \sqrt{a + b x^2 + c x^4}\right) + \left(a^{1/4} \left(b + 2 \sqrt{a} \sqrt{c}\right) \left(5 c d - 2 b f + 3 \sqrt{a} \sqrt{c}\, f\right) \left(\sqrt{a} + \sqrt{c}\, x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c}\, x^2\right)^2}}\; \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(30 c^{7/4} \sqrt{a + b x^2 + c x^4}\right)$$

Result (type 4, 1534 leaves):

$$\sqrt{a + b x^2 + c x^4} \left(\frac{6 b c e - 3 b^2 g + 8 a c g}{48 c^2} + \frac{\left(5 c d + b f\right) x}{15 c} + \frac{\left(6 c e + b g\right) x^2}{24 c} + \frac{f x^3}{5} + \frac{g x^4}{6}\right) + \frac{1}{240 c^2} \left(\left(\left(20 i \sqrt{2}\, b c \left(-b + \sqrt{b^2 - 4 a c}\right) d \sqrt{1 - \frac{2 c x^2}{-b - \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b - \sqrt{b^2 - 4 a c}}}\, x\right], \frac{-b - \sqrt{b^2 - 4 a c}}{-b + \sqrt{b^2 - 4 a c}}\right]\right)\right)\right) \Big/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\Bigg(8\,i\,\sqrt{2}\,b^2\left(-b+\sqrt{b^2-4\,a\,c}\right)f\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(24\,i\,\sqrt{2}\,a\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)f\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\Bigg(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)-\Bigg(80\,i\,\sqrt{2}\,a\,c^2\,d\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}$$

$$\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\right.$$

$$\left.\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\Bigg(8\,i\,\sqrt{2}\,a\,b\,c\,f\,\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\;\sqrt{a+b\,x^2+c\,x^4}\right)-15\,b^2\,\sqrt{c}\;e\,\text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\;\sqrt{a+b\,x^2+c\,x^4}\right]+$$

$$60\,a\,c^{3/2}\,e\,\text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\,\right]+\frac{15\,b^3\,g\,\text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\,\right]}{2\,\sqrt{c}}-$$
$$\left.30\,a\,b\,\sqrt{c}\,g\,\text{Log}\left[b+2\,c\,x^2+2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}\,\right]\right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 359 leaves, 8 steps):

$$\frac{g\,\sqrt{a+b\,x^2+c\,x^4}}{2\,c}+\frac{f\,x\,\sqrt{a+b\,x^2+c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{(2\,c\,e-b\,g)\,\text{ArcTanh}\left[\frac{b+2\,c\,x^2}{2\,\sqrt{c}\,\sqrt{a+b\,x^2+c\,x^4}}\right]}{4\,c^{3/2}}-$$
$$\left(a^{1/4}\,f\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$
$$\left(c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}\right)+\left(a^{1/4}\,\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+f\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(2\,c^{3/4}\,\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 526 leaves):

$$\frac{1}{4 c^{3/2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}} \left(i \sqrt{2} \sqrt{c} \left(-b+\sqrt{b^2-4 a c}\right) f \sqrt{\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right] - i \sqrt{2} \sqrt{c} \left(2 c d+\left(-b+\sqrt{b^2-4 a c}\right) f\right) \sqrt{\frac{b-\sqrt{b^2-4 a c}+2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right] + \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \left(2 \sqrt{c} g \left(a+b x^2+c x^4\right) + (2 c e-b g) \sqrt{a+b x^2+c x^4} \text{Log}\left[b+2 c x^2+2 \sqrt{c} \sqrt{a+b x^2+c x^4}\right]\right)\right)$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x+f x^2+g x^3}{\left(a+b x^2+c x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 447 leaves, 7 steps):

$$\frac{x \left(b^2 d-2 a c d-a b f+c (b d-2 a f) x^2\right)}{a \left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}} - \frac{b e-2 a g+(2 c e-b g) x^2}{\left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}} - \frac{\sqrt{c} (b d-2 a f) x \sqrt{a+b x^2+c x^4}}{a \left(b^2-4 a c\right) \left(\sqrt{a}+\sqrt{c} x^2\right)} + \left(c^{1/4} (b d-2 a f) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(a^{3/4} \left(b^2-4 a c\right) \sqrt{a+b x^2+c x^4}\right) - \left(\left(\sqrt{c} d-\sqrt{a} f\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(2 a^{3/4} \left(b-2 \sqrt{a} \sqrt{c}\right) c^{1/4} \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 513 leaves):

$$-\frac{1}{4\,a\,\left(b^2-4\,a\,c\right)\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{a+b\,x^2+c\,x^4}}$$

$$\left(4\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\left(-2\,a^2\,g-b\,d\,x\,\left(b+c\,x^2\right)+2\,a\,c\,x\,\left(d+x\,\left(e+f\,x\right)\right)+a\,b\,\left(e+x\,\left(f-g\,x\right)\right)\right)+\right.$$

$$i\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\left(b\,d-2\,a\,f\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$i\,\left(-b^2\,d+4\,a\,c\,d+b\,\sqrt{b^2-4\,a\,c}\,d-2\,a\,\sqrt{b^2-4\,a\,c}\,f\right)$$

$$\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x+f\,x^2+g\,x^3}{\left(a+b\,x^2+c\,x^4\right)^{5/2}}\,dx$$

Optimal (type 4, 680 leaves, 9 steps):

$$\frac{x\left(b^2 d-2 a c d-a b f+c\left(b d-2 a f\right) x^2\right)}{3 a\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)^{3/2}}-\frac{b e-2 a g+\left(2 c e-b g\right) x^2}{3\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)^{3/2}}+\frac{4\left(2 c e-b g\right)\left(b+2 c x^2\right)}{3\left(b^2-4 a c\right)^2\sqrt{a+b x^2+c x^4}}+$$

$$\left(x\left(2 b^4 d-17 a b^2 c d+20 a^2 c^2 d+a b^3 f+4 a^2 b c f+c\left(2 b^3 d-16 a b c d+a b^2 f+12 a^2 c f\right) x^2\right)\right)\Big/$$

$$\left(3 a^2\left(b^2-4 a c\right)^2\sqrt{a+b x^2+c x^4}\right)-\frac{\sqrt{c}\left(2 b^3 d-16 a b c d+a b^2 f+12 a^2 c f\right) x\sqrt{a+b x^2+c x^4}}{3 a^2\left(b^2-4 a c\right)^2\left(\sqrt{a}+\sqrt{c}\, x^2\right)}+$$

$$\left(c^{1/4}\left(2 b^3 d-16 a b c d+a b^2 f+12 a^2 c f\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(3 a^{7/4}\left(b^2-4 a c\right)^2\sqrt{a+b x^2+c x^4}\right)-$$

$$\left(c^{1/4}\left(2 b^2 d-3\sqrt{a}\, b\sqrt{c}\, d-10 a c d+a b f+6 a^{3/2}\sqrt{c}\, f\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\right.$$

$$\left.\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(6 a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(b^2-4 a c\right)\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 598 leaves) :

$$\frac{1}{12\, a^2\, (b^2-4\, a\, c)^2\, (a+b\, x^2+c\, x^4)^{3/2}}$$

$$\Bigg(-4\, a\, (b^2-4\, a\, c)\, \left(-2\, a^2\, g-b\, d\, x\, (b+c\, x^2)+2\, a\, c\, x\, (d+x\, (e+f\, x))+a\, b\, (e+x\, (f-g\, x))\right)+$$

$$4\, (a+b\, x^2+c\, x^4)\, \Big(2\, b^3\, d\, x\, (b+c\, x^2)+a\, b\, x\, (-17\, b\, c\, d+b^2\, f-16\, c^2\, d\, x^2+b\, c\, f\, x^2)+$$
$$4\, a^2\, \left(-b^2\, g+c^2\, x\, (5\, d+x\, (4\, e+3\, f\, x))+b\, c\, (2\, e+x\, (f-2\, g\, x))\right)\Big)+$$

$$\frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4\, a\, c}}}}\, \mathbb{i}\, \sqrt{2}\, \sqrt{\frac{b+\sqrt{b^2-4\, a\, c}+2\, c\, x^2}{b+\sqrt{b^2-4\, a\, c}}}\, \sqrt{1+\frac{2\, c\, x^2}{b-\sqrt{b^2-4\, a\, c}}}\, (a+b\, x^2+c\, x^4)$$

$$\Bigg(-\left(-b+\sqrt{b^2-4\, a\, c}\right)\, (2\, b^3\, d-16\, a\, b\, c\, d+a\, b^2\, f+12\, a^2\, c\, f)$$

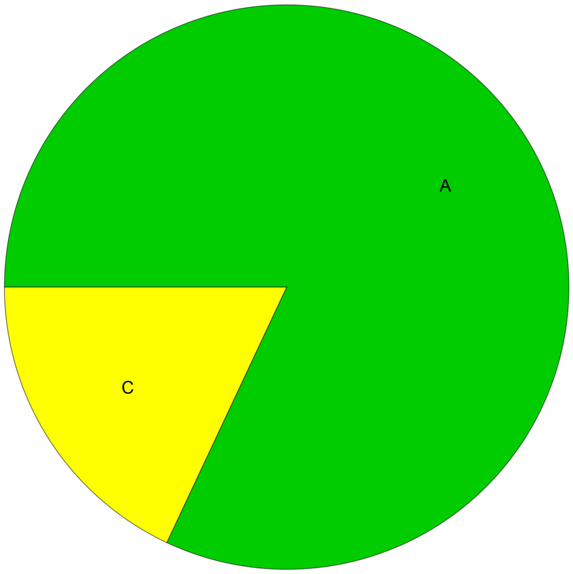
$$\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\sqrt{2}\, \sqrt{\frac{c}{b+\sqrt{b^2-4\, a\, c}}}\, x\right],\, \frac{b+\sqrt{b^2-4\, a\, c}}{b-\sqrt{b^2-4\, a\, c}}\right]+$$

$$\Big(-2\, b^4\, d+b^3\, \left(2\, \sqrt{b^2-4\, a\, c}\, d-a\, f\right)+4\, a\, b\, c\, \left(-4\, \sqrt{b^2-4\, a\, c}\, d+a\, f\right)+$$
$$a\, b^2\, \left(18\, c\, d+\sqrt{b^2-4\, a\, c}\, f\right)+4\, a^2\, c\, \left(-10\, c\, d+3\, \sqrt{b^2-4\, a\, c}\, f\right)\Big)$$

$$\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\sqrt{2}\, \sqrt{\frac{c}{b+\sqrt{b^2-4\, a\, c}}}\, x\right],\, \frac{b+\sqrt{b^2-4\, a\, c}}{b-\sqrt{b^2-4\, a\, c}}\right]\Bigg)\Bigg)$$

Summary of Integration Test Results

111 integration problems



A - 91 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 20 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts